Algebraic Number Theory Mid-Term

March 5 2014

Please do not cheat. Good Luck! This exam is of 30 marks. There are 6 questions. (30)

1. Prove that the set of algebraic numbers $\mathbb{A}$ is a field. (4)

2a. Compute a discriminant of $\mathbb{Q}(\sqrt[3]{2})$. (3)
2b. Compute **the** discriminant of $\mathbb{Q}(\sqrt{21})$. (3)

3. Give an example of a ring R with the following properties — if possible. If not, explain why.

- Not every irreducible is prime. (2)
- Not every prime is irreducible. (2)
- Not every prime ideal is generated by at most 2 elements. (2)
- A UFD which is not a PID. (1)

4. Is the group of units in $\mathbb{Z}(\sqrt{5})$ finite or infinite? Why? (3)

5a. Factorize 18 in $\mathbb{Z}[\sqrt{-17}]$ in as many ways as possible. (3)
5b. Factorize (18) in $\mathbb{Z}[\sqrt{-17}]$ in to prime ideals. (3)

6. Find all ideals in $\mathbb{Z}[\sqrt{2}]$ containing the ideal
a. (7) (2)
b. (5) (2)